## Efficiently Evaluating

Complex Boolean
Expressions
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## Agenda

- Motivation and problem definition
- Algorithms
- Experiments


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## Simple example

- Display advertising
- Ads: Boolean expressions (contracts) age IN \{young\}
age IN \{old\} AND income IN \{high, veryHigh\}
income IN \{high\} AND browser NOT_IN \{ie\}
- Publishers: assignments
age $=$ old; income $=$ high; browser $=$ firefox


## Simple example

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- Ads: Boolean expressions (contracts) age IN \{young\}
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## More complex example

- Display advertising exchange



## More complex example

- Boolean expressions model the type of inventory sold by each node



## More complex example

- Each Boolean expression can be a DNF/CNF
- Contracts for the publisher are "complex" expressions



## Other examples

- Automatic targeting in display advertising
- e.g. machine generated expressions to maximize click-through
- Information dissemination in social network graphs


## State-of-the-art

- There are existing solutions for efficiently evaluating CNF and DNF expressions
- Content-based publish-subscribe systems
- Normalizing complex Boolean expressions into DNF incurs in an exponential blow-up in size


## DNF growth

- In KB, averaged over 20 DNFs of each size
- Data set is realistic



## Problem definition

- Evaluate complex Boolean expressions (e.g.AND of DNFs)
- Modeled as a tree of AND/OR nodes, where leafs are conjunctions of IN and NOT_IN operators
- Given an assignment, retrieve all valid expressions


## Intuition

- (Offline) Annotate the conjunctions with their position on the complex Boolean expression tree
- Evaluate conjunctions (leafs) using a state-of-the-art algorithm
- Evaluate the trees bottom-up, using the retrieved conjunctions and their positions


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## Online problem

- Given a set of valid conjunctions, is the Boolean expression satisfied
- Tree is never explicitly represented



## Algorithm I: Dewey ids

- Assign Dewey ids for every node in the expression tree
- Ordering children of a node



## Algorithm I: Dewey ids

- Alternating AND/OR trees
-     * denotes last child of an AND node



## Algorithm I: Dewey ids

- Index evaluator will return the leaf nodes, which are the matching conjunctions

$$
\begin{aligned}
& \square 1.2 \\
& \text { I.I.I.I } \quad 2^{*} .1 .1 .2 \quad 2^{*} .3 .1 .1 \quad 2^{*} .3 .2^{*} .1
\end{aligned}
$$

## Algorithm I: Dewey ids

- Index evaluator will return the leaf nodes, which are the matching conjunctions



## Algorithm 2: Interval ids

- We map each Boolean tree to a one dimensional interval [I,M]
- $M$ is the maximum number of leaves
- Tree is satisfied if there is a subset of intervals that cover all integer points on [I,M] without overlap


## Algorithm 2: Interval ids

- Look at: [I-5] [6-I4] [I5-M] : all integer points covered without overlap



## Assigning intervals

- Recursive procedure
- Children of an OR inherit the parent interval



## Assigning intervals

- Recursive procedure
- Children of an AND partition the interval


$$
12345678910
$$



# Slightly more complex example 

- B \& D are not enough to satisfy, since intervals overlap
- D \& E \& F are OK, since intervals don't overlap



B 2345678910

| C 12345678910 |
| :--- | :--- |

D 1234


## Example

- Suppose intervals returned are
- [I, I], [I,4], [5,5], [6, I0]
- Final matched array: I \| 0 0 I \| 0 0 0 0 I



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## Data

- Generated a synthetic data set of expressions based on real logs
- Depth of the tree between I and 4
- Typical number of children of nodes between I and 4


# Performance of different methods 

- Running time in ms (y axis) vs. tree depth (x axis). Scan does not scale wrt time



## DNF performance

- Running time in ms (y axis) vs. tree depth (x axis)



## Interval and Dewey

- Running time of the tree evaluation in ms (y axis) vs. \#boolean expressions in test



## Conjunction matching time

- Running time of the tree evaluation in ms (y axis) vs. tree depth



## Excluding conjunction matching



